Compact Stabilized Semiconductor Laser for Frequency Metrology

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ABSTRACT

We report on the development of a frequency modulatable 795 nm semiconductor laser based on self-injection locking to a high quality factor whispering gallery mode microresonator. The laser is characterized with residual amplitude modulation below -80 dB and frequency noise better than 300 Hz/Hz$^{1/2}$ at offset frequencies ranging from 100 Hz to 10 MHz. The frequency modulation (FM) speed and span of the laser exceed 1 MHz and 4 GHz, respectively. Locking of the laser to Doppler-free saturated absorption resonance of ^{87}Rb D$_1$ line is demonstrated and frequency stability below 10^{-12} is measured for integration time spanning from 1 s to 1 day. The architecture demonstrated in this study is suitable for realization of frequency modulatable lasers at any wavelength.

Keywords: Lasers, Tunable Lasers, Frequency Modulated Lasers, High-Q Resonators, Laser Stabilization, Whispering Gallery Modes

1. INTRODUCTION

Laser wavelength modulation (WM)[1] and frequency modulation (FM)[2,3] spectroscopy is a versatile technique to selectively detect variations of optical absorption caused by atomic and molecular transitions. This approach is widely utilized in a variety of applications, including FM spectroscopy, and atomic clocks. In these and related applications the signal contrast is an important parameter since it determines the ultimate signal to noise ratio, which is a key factor in the achievable performance. The sensitivity limit of WM/FM spectroscopy is also controlled by the background signals originating from residual amplitude modulation (RAM)[4–13] as well as the inherent noise of the laser, which sets the frequency resolution limit.

The detection of low contrast absorption features in FM spectroscopy is aided by fast modulation of frequency at rates comparable with the bandwidth of the resolved features.[14] The low contrast spectroscopic signal is extracted by self-homodyning lock-in methods, which improve the signal-to-noise ratio by essentially subtracting the contribution of base-band amplitude and frequency noise of the laser. Resolution improvements of 40 dB and more are easily obtained by combining high-performance stabilized lasers with external frequency or phase (acousto-optic and electro-optic) modulators. Externally modulated FM spectroscopy systems can resolve parts per million sub-Doppler and multiphoton absorption features, reaching sub-Hz resolution to achieve the high level of signal-to-noise ratio that is important in optical frequency metrology, pectroscopy, and optical clocks.

Semiconductor lasers are attractive for these applications,[15–19] especially where compact spectroscopy systems are desired. Diode lasers are available at many optical wavelengths and are directly tunable/ditherable via either modulation of their injection current or thier temperature. Resolution, wavelength precision, and sensitivity of diode laser-based FM spectroscopy is limited by their relatively broad linewidth ($\approx$ 1 MHz), high frequency noise, as well as the significant RAM of the laser. This occurs in spite of high rates and spans of modulation that are achievable with direct modulation of Fabry-Perot (FP) and distributed feedback (DFB) lasers. The linewidth can be improved and frequency modulation spans can by increased and RAM can be reduced by using external cavity frequency stabilization technique. The optimized laser linewidth achieved with this approach is on the order of tens of kHz in the common external cavity diode lasers used in Littman or Littrow configurations.[20–25] It is uncommon to achieve a combination of a narrow linewidth (1-10 kHz), broad dithering span (exceeding 1 GHz), and high dithering frequency (10 kHz and higher rates) desirable for many spectroscopic applications.

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In this work we report on demonstration of a semiconductor laser with substantially better characteristics, as compared with the bare diodes or conventional extended cavity diode lasers, that allow a significant increase in the accuracy and the sensitivity of the spectroscopic measurements. In particular, FM modulation rate of our laser is above 1 MHz, its modulation span is above 4 GHz, and the measured RAM, at -80 dB, is at least an order of magnitude lower than other FM modulated devices known to us.

To achieve these improvements, we built an external cavity distributed feedback (DFB) semiconductor laser using a high quality (Q-) factor monolithic whispering gallery mode (WGM) microresonator.[26] The modulating and dithering of the laser frequency was produced by tuning the external cavity. The frequency of the WGM resonator was modified by changing its temperature as well by stress applied via a piezo-electric transducer (PZT) actuator (see Fig. 1).

We utilized theself-injection locking method for locking the laser to the WGM resonator.[27–32] In this scheme the resonant stimulated Rayleigh back-scattering is used, which occurrs at the frequency of a high-Q resonator mode due to surface and volumetric inhomogeneities in the resonator.[33,34] Because self-injection locking feedback is rather fast, it results in significant reduction of the laser phase and, to a lesser extent, amplitude noise, in a broad frequency range.[35–37] It also allows transferring frequency modulation from the resonator to the laser. The advantage of this transfer is that the modulation is not associated with the change of the laser power and, hence, the RAM effect is extremely small. We achieved RAM below -80 dB, which is orders of magnitude smaller than RAM with electro-optical modulators.[12,13] The achieved value of RAM is in contrast to that produced by direct modulation of a DFB laser through the change of the carrier density, which unavoidably leads to a significant admixture of amplitude modulation in the phase modulated signal.

We illustrated the performance of the laser by locking it to a saturated absorption transition in rubidium (Fig. 1) and demonstrated frequency stability exceeding 10^{-12} for integration times from 1 s to 1 day. The achieved stability of the locked laser, combined with its compact package (50 cm^3) makes it useful for application in atomic clocks, magnetometers, wavelength references and other high precision applications based on frequency metrology.

2. RAM AND LASER SPECTROSCOPY

Frequency modulation and wavelength modulation laser spectroscopy[1–3,14] benefits from usage of heterodyne methods to shift the frequency of the measured signal from the low frequency region where technical noise as well as drifts dominate, to higher frequencies. However, residual amplitude modulation of light used in the spectroscopy mimics the signal of interest, leading to a nonzero background that limits the measurement sensitivity. In what follows we present a simplified explanation of the RAM's impact following earlier published studies.[14]

2.1 Frequency modulation spectroscopy

Let us consider an FM spectroscopy scheme involving a probe laser characterized with excessive intensity noise. For the sake of clarity, we consider the problem of determination of the center of a resonance having less than 100% contrast. The complex amplitude transfer function of such a resonance can be described by the expression

$$F(\omega) = \frac{E_{out}}{E_{in}} = \frac{\gamma_1 - \gamma_2 + i(\omega_0 - \omega)}{\gamma_1 + \gamma_2 + i(\omega_0 - \omega)}, \tag{1}$$

where ω is the spectral frequency, ω_0 is the frequency of the resonance, E_{in} and E_{out} are the complex amplitudes of the electric field of the input and output light waves, γ_1 and γ_2 are the parameters determining contrast, $1 \geq C \geq 0$, defined as

$$C = 1 - \left.\frac{|E_{out}|^2}{|E_{in}|^2}\right|_{\omega=\omega_0} = \frac{4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}, \tag{2}$$

and bandwidth

$$\Gamma = \gamma_1 + \gamma_2 \tag{3}$$

of the resonance.

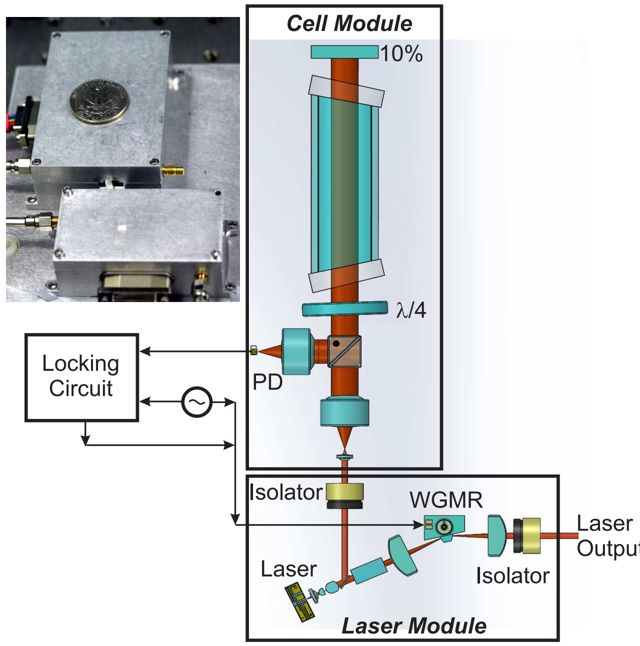

Figure 1. Schematic of the experimental setup including the self-injection locked modulatable laser and a rubidium cell (the ratio between geometrical sizes of the components is conserved). Inset shows actual assembled prototype. Light from a semiconductor DFB laser (30 mW) is collimated and injected to a modulatable WGM microresonator (WGMR). The coupling efficiency is 3 dB. There is no isolator between the laser and the resonator so the laser self-injection locks to a resonator mode. Approximately 0.5 mW is split off the laser beam, expanded, and injected to the rubidium vapor cell through a polarizing beam splitter (PBS) and a quarter wave plate ($\lambda/4$). Ten percent of the light is reflected back to generate the saturated absorption resonance used to lock the device. The light exiting the cell is analyzed using photodiode PD. Signal from the photodiode is used to create an electronic feedback stabilizing the laser. The laser has 10 mW output.

The input light is modulated to generate an error signal. We assume that both the amplitude and phase modulation are present, and that the modulation is small

$$E_{in} = E_0 \left(1 + a\cos\Omega t\right) e^{-i(\omega t + b\sin\Omega t)} \approx \\ E_0 e^{-i\omega t}\left[1 + \left(\frac{a}{2} - \frac{b}{2}\right) e^{i\Omega t} + \left(\frac{a}{2} + \frac{b}{2}\right) e^{-i\Omega t}\right]. \quad (4)$$

For the amplitude of light that interacted with the resonance we find

$$E_{out} = E_0 e^{-i\omega t}\left[F(\omega) + \left(\frac{a}{2} - \frac{b}{2}\right) F(\omega - \Omega) e^{i\Omega t} + \\ \left(\frac{a}{2} + \frac{b}{2}\right) F(\omega + \Omega) e^{-i\Omega t}\right]. \quad (5)$$

Let us assume now that the modulated light is sent to a photodiode with resistance ρ and responsivity R, so the photocurrent i_{PD} is equal to

$$i_{PD} = RP_{out}, \quad (6)$$

where P_{out} is the optical power at the photodiode (PD). We are interested in heterodyne detection of the first modulation harmonic of the exiting RF signal

$$\left.\frac{P_{out}}{P_0}\right|_{\exp(\pm i\Omega t)} = \frac{1}{2}\left[(a-b)F(\omega)F^*(\omega - \Omega) + \\ (a+b)F^*(\omega)F(\omega + \Omega)\right] e^{-i\Omega t} + c.c. \quad (7)$$

To measure it, the photocurrent is electronically mixed with a local oscillator signal

$$i_{LO} \sim e^{i(\Omega t+\phi_{LO})} + e^{-i(\Omega t+\phi_{LO})}, \tag{8}$$

and filtered out with a low pass filter of bandwidth ΔF, to produce a DC error signal with power

$$P_{error} = \rho R^2 P_0^2 \frac{\mu}{4} \{[(a-b)F(\omega)F^*(\omega-\Omega)+ \\ (a+b)F^*(\omega)F(\omega+\Omega)]\, e^{i\phi_{LO}} + c.c.\}^2, \tag{9}$$

where μ is the mixer efficiency.

We assume that the modulation frequency is small compared with the spectral width of the resonance, $\Gamma \gg \Omega$. In this case the expression in square brackets in Eq. (9) is real and, hence, we have to select $\phi_{LO} = 0$. The error signal becomes

$$P_{error} = \rho R^2 P_0^2 \frac{\mu}{4} \left\{ 4a \frac{(\gamma_1-\gamma_2)^2 + (\omega_0-\omega)^2}{(\gamma_1+\gamma_2)^2 + (\omega_0-\omega)^2} - \\ 16b \frac{\gamma_1\gamma_2(\omega_0-\omega)\Omega}{((\gamma_1+\gamma_2)^2 + (\omega_0-\omega)^2)^2} \right\}^2, \tag{10}$$

We obtain the first important result of the calculation from Eq. (10): the amplitude modulation shifts the zero of the error signal by δ_{AM}, that can be written in the form

$$\frac{\delta_{AM}}{\Gamma} = \frac{a}{b}\frac{1-C}{C}\frac{\Gamma}{\Omega}. \tag{11}$$

It means that even small admixture of amplitude modulation results in a significant shift of the zero of the error signal if i) Ω is much smaller than Γ and, ii), the contrast is small, $C < 1$.

The second important result is that the error signal is given by

$$P_{error} = 4\mu\rho R^2 P_0^2 b^2 C^2 \left(\frac{\omega_0-\omega}{\Gamma}\right)^2 \left(\frac{\Omega}{\Gamma}\right)^2. \tag{12}$$

If we note that the power of the first modulation sideband is approximately $P_1 = P_0 b^2/4$, we get

$$P_{error} = 16\mu\rho R^2 P_0 P_1 C^2 \left(\frac{\omega_0-\omega}{\Gamma}\right)^2 \left(\frac{\Omega}{\Gamma}\right)^2. \tag{13}$$

Let us assume that the major noise that limits the sensitivity is the relative intensity noise (RIN) at the modulation frequency. The power of the RIN-originated RF noise is

$$P_{RIN} = \mu\rho R^2 P_0^2 (1-C)^2 RIN \Delta F. \tag{14}$$

We represent the signal to noise ratio (SNR) as P_{error}/P_{RIN}, and assuming that the accuracy of the lock is given by $SNR = 1$, find

$$\frac{|\omega_0-\omega|}{\Gamma} \approx \sqrt{\frac{P_0}{P_1}}\frac{1-C}{4C}\frac{\Gamma}{\Omega}\sqrt{RIN(\Omega)\Delta F}. \tag{15}$$

2.2 Wavelength modulation spectroscopy

Similar result can be obtained for the case of wavelength modulation spectroscopy for which frequency deviation exceeds the modulation frequency. The phase term of the frequency modulated light can be presented in this case as

$$\exp -i \int_0^t (\omega + \Omega_m \cos\omega_m \tau) d\tau, \tag{16}$$

where Ω_m is the frequency deviation and $\omega_m \ll \Omega_m$ is the modulation frequency. We cannot use the simple harmonic decomposition utilized in Eq. (5) and instead use the following expression

$$E_{in} = E_0 \left(1 + a \cos \omega_m t\right) \exp\left[-i \left(\omega t + \frac{\Omega_m}{\omega_m} \sin \omega_m t\right)\right]. \tag{17}$$

The error signal is given in this case by the derivative of the absorption profile of the line[14]

$$P_{error} \simeq \rho R^2 P_0^2 \mu \left(2a + \frac{d|F(\omega)|^2}{d\omega}\Omega_m\right)^2 \simeq \tag{18}$$
$$\rho R^2 P_0^2 \mu \left[2a + 8\frac{\gamma_1\gamma_2(\omega_0 - \omega)\Omega_m}{((\gamma_1+\gamma_2)^2 + (\omega_0-\omega)^2)^2}\right]^2.$$

The main approximation here is that the frequency deviation is much less than the bandwidth of the spectral line under study $\Gamma \gg \Omega_m$. The error signal decreases when Ω_m exceeds Γ.[14]

Therefore, the center of the error signal shifts due to AM modulation as

$$\frac{\delta_{AM}}{\Gamma} = a\frac{1-C}{C}\frac{\Gamma}{\Omega_m}, \tag{19}$$

and the SNR limited measurement accuracy of the center of the resonance is

$$\frac{|\omega_0 - \omega|}{\Gamma} \approx \frac{1-C}{2C}\frac{\Gamma}{\Omega_m}\sqrt{RIN(\Omega)\Delta F}. \tag{20}$$

Comparing Eq. (11) and Eq. (15) as well as Eq. (19) and Eq. (20) we conclude that if $\sqrt{RIN(\Omega)\Delta F} \gg a$ the AM modulation term does not influence the measurement accuracy, within the frame of the approximations made, . This is not the case for the majority of diode laser spectroscopy experiments. Therefore, the amplitude modulation term has to be reduced as much as possible to increase the measurement accuracy.

3. EXPERIMENT

In our experiment we created two identical self-injection locked lasers using WGM resonators having a diameter of 2 mm and loaded bandwidth of 500 kHz (Fig. 1). The resonator was modulated using a PZT actuator. The PZT stresses the microresonator modifying its diameter as well as creating a change in the refraction index at the location of the optical mode through the elasto-optic effect. These effects cause the frequency of the WGMs to change. The laser tracks the changing frequency of the optical mode when self-injection locked, creating high speed frequency modulation in the optical output with very low residual amplitude noise. The measured modulation rate was 10 MHz/V, with respect to the voltage at the PZT element.

To measure the FM response of the laser, we coupled the output light into a single mode fiber and send it through a frequency discriminator made of a Mach-Zehnder Interferometer (MZI), which converts frequency modulation to intensity modulation. A network analyzer (3577A) was used to measure the modulation frequency response. To detect the RAM associated with the FM signal we sent the light to a Thorlabs photodiode and performed the measurement using the same network analyzer. The phase and amplitude of the FM response, and the amplitude of the RAM are shown in Fig. (2). It worth noting that there are several spurs in the PZT response above 100 kHz, and the data in Fig. (2) does not have enough resolution bandwidth to see them.

To further characterize the performance of the lasers we measured their frequency noise, Fig. (3), as well as RIN, Fig. (4). We introduced the output of both lasers toa fast photodiode and prroduced a beat signal. The beat-note radio frequency (RF) signal was centered around 10 GHz and its phase noise was measured using the RF phase noise test system (PNTS) of OEwaves. With the optical power on the photodiode at 3 mW, and the photodiode responsivity of 0.3 A/W, the output RF power was -17 dBm. The phase noise was converted into frequency noise and shown in Fig. (3).

To measurethe laser RIN we sent the light of one laser to a fast photodiode and measured the signal using a signal analyzer. We found that self-injection locking results in improvement of the RIN if the laser is resonantly tuned to the corresponding WGM. The RIN can increase if the laser is locked to the slope of the mode. This effect results from the FM-to-AM conversion via the resonator which is effectively a frequency discriminator.

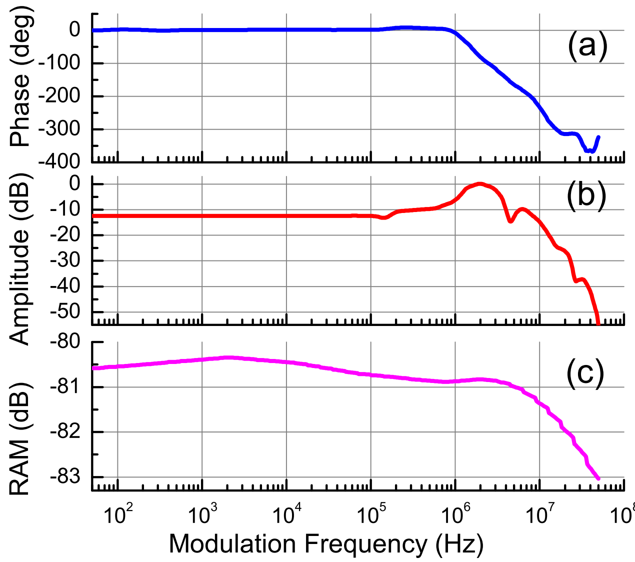

Figure 2. Modulation properties of the laser.

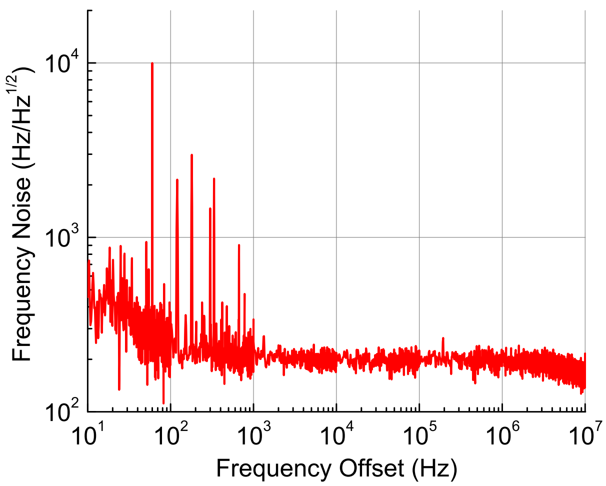

Figure 3. Frequency noise of a free running self-injection locked laser.

3.1 Locking FM lasers to rubidium transitions

To demonstrate the suitability of the laser for FM spectroscopy, we locked the lasers to neighboring saturation absorption resonances of D_1 line og Rb and measured Allan deviation of the beat note of the lasers (see Fig. 1). We used two independent pure ^{87}Rb isotope cells (Triad Technology), with dimensions of 25 mm long and 10 mm diameter. The cells were kept at 38 C^o using a slow surface heaters (Minco). They were also shielded against the ambient magnetic fields using a single layer μ-metal sheet.

The cell windows were wedged and anti-reflection coated to reduce back-reflection. The laser light first was sent through a polarizing beam splitter (PBS) and only π-polarized light was transmitted. It then passed a quarter-wave plate before entering the cell. The light was partially reflected (10%) at the back of the cell and went back along the same path as the incoming light. The light exiting the cell passed through the quarter-wave plate again and became σ-polarized. It was then reflected by the PBS and focused on a photodiode, which detected the saturated absorption signal needed for locking of the laser.

The laser beam in the cell had 2.8 mm in diameter and the interrogation power was 0.5 mW. At temperature of 38 C^o,the resultant saturated absorbtion resonances had 20% contrast. The lasers were frequency modulated at 100 kHz with modulation span of 2 MHz and locked to the atomic transition using the Pound-Drever-Hall

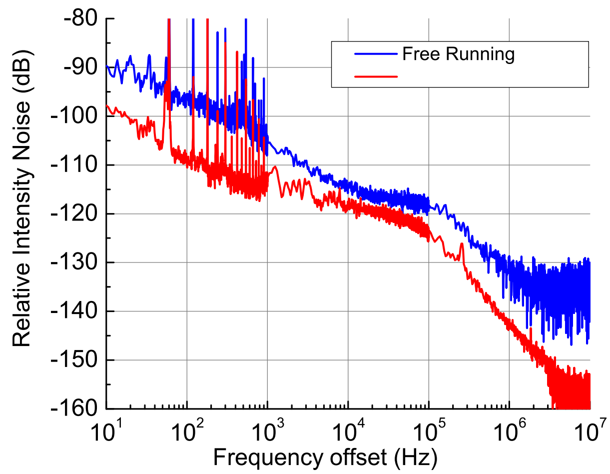

Figure 4. Comparison of RIN of free running DFB laser and self-injection locked DFB laser. The noise decreases in the self-injection locked case.

technique.[39] The error signals was generated from a SRS-830 lock-in amplifier and processed by a SRS PID controller. It was then combined with the modulation signal via a bias-T and fed back to the WGM PZT modulation input.

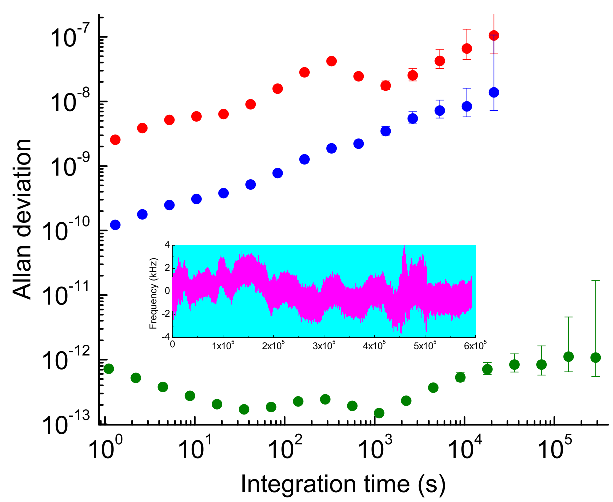

Figure 5. Allan deviation of the free running DFB laser (red dots), self-injection locked DFB laser (blue dots), and self-injection locked DFB laser locked to the Doppler free rubidium transition separated by 812 MHz (green dots). The inset shows measurement trace of the frequency of the lasers' beatnote versus time that is characterized with Allan deviation shown by green dots.

The emission from the lasers was sent to a fast photodiode that generated an RF signal, stability of which was studied using a frequency counter slaved to a commercial rubidium clock. The result of the measurement is shown at Fig. (5) by green dots. The stability of the lasers' was better than 10^{-12} within the measurement interval of the expriment, as shown. We also repeated the measurement for the free running DFB lasers as well as the DFB lasers self-injection locked to WGM resonators. The corresponding Alan deviation is illustrated by red and blue dots in Fig. (5). Therefore, the self-injection locking along with the electronic locking to the saturated absorption resonances of rubidium allowed improving the stability of the lasers by more than four orders of magnitude.

The stability reported in this work is higher than the stability observed with a previously demonstrated compact extended cavity diode laser locked to D_2 line of atomic ^{87}Rb.[40,41] It is also comparable with the stability achieved in a laboratory scale diode laser setup involving Rb saturated absorption spectroscopy.[42] Interestingly, the excellent stability of our lasers was achieved without fast active stabilization of the cells' temperature, which was varying by approximately 0.3 C^o due to air-conditioner cycle having ≈1000 s period.

4. CONCLUSION

We have demonstrated a frequency modulated diode laser with reduced relative amplitude modulation. The laser allows accurate locking to rubidium saturated absorption line. We demonstrated stability of the locking exceeding 10^{-12} at time intervals from one second to a day. Since the modulation technology is rather generic, it can be applied to any laser self-injection locked to a WGM resonator, expanding usability of the lasers for accurate FM spectroscopy.

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